

## Problem 2.11

[Difficulty: 3]

**2.11** The flow field for an atmospheric flow is given by

$$\vec{V} = -\frac{My}{2\pi}\hat{i} + \frac{Mx}{2\pi}\hat{j}$$

where  $M = 1 \text{ s}^{-1}$ , and the  $x$  and  $y$  coordinates are the parallel to the local latitude and longitude. Plot the velocity magnitude along the  $x$  axis, along the  $y$  axis, and along the line  $y = x$ , and discuss the velocity direction with respect to these three axes. For each plot use a range  $x$  or  $y = 0$  km to 1 km. Find the equation for the streamlines and sketch several of them. What does this flow field model?

**Given:** Flow field

**Find:** Plot of velocity magnitude along axes, and  $y = x$ ; Equation for streamlines

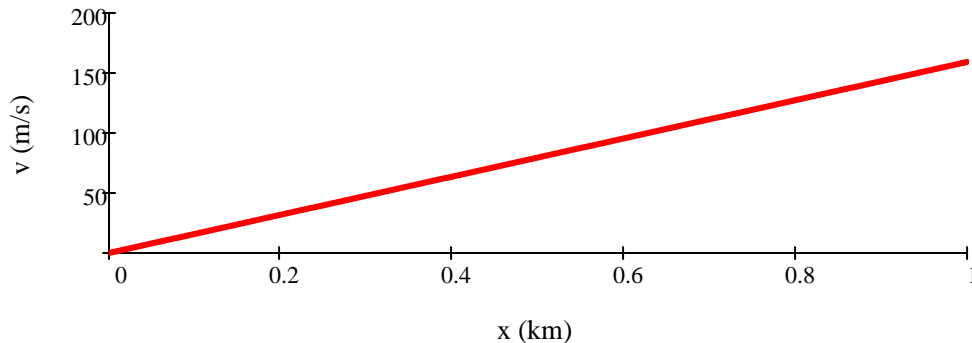
**Solution:**

On the  $x$  axis,  $y = 0$ , so

$$u = -\frac{M \cdot y}{2 \cdot \pi} = 0$$

$$v = \frac{M \cdot x}{2 \cdot \pi}$$

Plotting



The velocity is perpendicular to the axis and increases linearly with distance  $x$ .

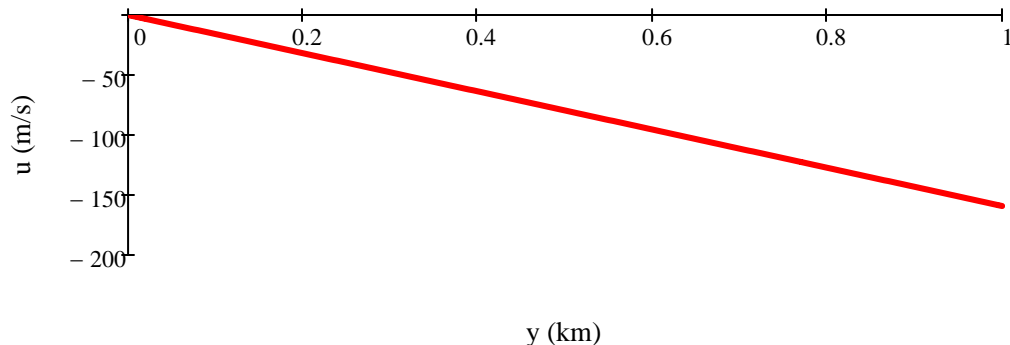
This can also be plotted in Excel.

On the  $y$  axis,  $x = 0$ , so

$$u = -\frac{M \cdot y}{2 \cdot \pi}$$

$$v = \frac{M \cdot x}{2 \cdot \pi} = 0$$

Plotting



The velocity is perpendicular to the axis and increases linearly with distance  $y$ .

This can also be plotted in Excel.

On the  $y = x$   
axis

$$u = -\frac{M \cdot y}{2 \cdot \pi} = -\frac{M \cdot x}{2 \cdot \pi} \quad v = \frac{M \cdot x}{2 \cdot \pi}$$

The flow is perpendicular to line  $y = x$ :

Slope of line  $y = x$ : 1

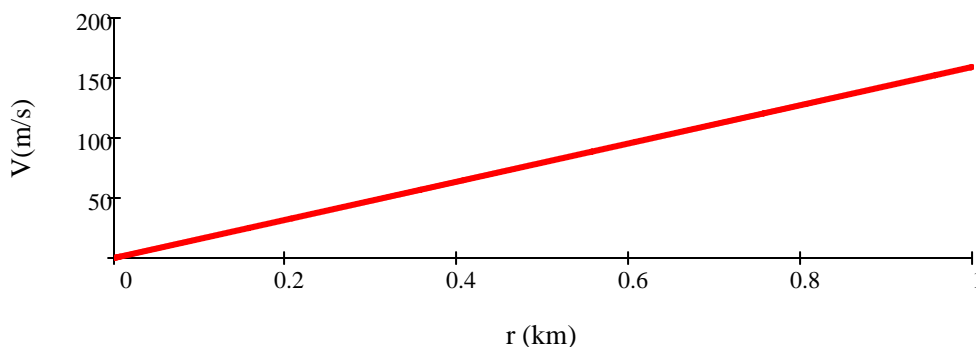
Slope of trajectory of motion:  $\frac{u}{v} = -1$

If we define the radial position:

$$r = \sqrt{x^2 + y^2} \quad \text{then along } y = x \quad r = \sqrt{x^2 + x^2} = \sqrt{2} \cdot x$$

$$\text{Then the magnitude of the velocity along } y = x \text{ is } V = \sqrt{u^2 + v^2} = \frac{M}{2 \cdot \pi} \cdot \sqrt{x^2 + x^2} = \frac{M \cdot \sqrt{2} \cdot x}{2 \cdot \pi} = \frac{M \cdot r}{2 \cdot \pi}$$

Plotting



This can also be plotted in  
Excel.

For  
streamlines

$$\frac{v}{u} = \frac{dy}{dx} = \frac{\frac{M \cdot x}{2 \cdot \pi}}{-\frac{M \cdot y}{2 \cdot \pi}} = -\frac{x}{y}$$

So, separating  
variables

$$y \cdot dy = -x \cdot dx$$

Integrating

$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

The solution  
is

$$x^2 + y^2 = C \quad \text{which is the equation of a circle.}$$

The streamlines form a set of concentric circles.

This flow models a rigid body vortex flow. See Example 5.6 for streamline plots. Streamlines are circular, and the velocity approaches zero as we approach the center. In Problem 2.10, we see that the streamlines are also circular. In a real tornado, at large distances from the center, the velocities behave as in Problem 2.10; close to the center, they behave as in this problem.